Distorting Attention: Platform Design, Market Complexity, and the Balance Between Advertising and Fees*

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Abstract

Digital retail platforms monetize both transactions and attention. This paper studies how a platform balances these margins when consumers face limited cognitive capacity. I develop a model in which buyers allocate costly attention before purchasing, and the platform chooses both a transaction fee and advertising intensity. Advertising generates revenue directly but also increases the difficulty of evaluating products, distorting consumer learning. The central mechanism is that advertising reshapes the information environment. In simple markets, consumers can easily assess quality, and advertising adds little value. In highly complex markets, further distortions discourage learning and reduce purchases. Advertising is therefore most profitable in markets of intermediate cognitive complexity. Transaction fees, in contrast, decline as complexity rises. While total profit falls with complexity, the advertising share of revenue is hump-shaped. Platforms that combine both instruments outperform ad-only and fee-only models, providing a unified framework for understanding the strategic design of digital markets.

JEL classification: D83, L13, L86, D47

Keywords: rational inattention; digital platforms; advertising load; cognitive complexity

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1 Introduction

Digital platforms (such as eBay) do not merely intermediate transactions; they design the informational environments in which decisions are made. For instance, advertising load, interface structure, or complex categorization can make it easier or harder for consumers to evaluate products. At the same time, consumers face attention constraints. Understanding how platforms exploit this constraint is central to modern industrial organization.

This paper studies how a platform optimally chooses both a transaction fee and an advertising intensity when consumers are rationally inattentive. Buyers must decide how much attention to devote before purchasing from sellers of uncertain quality. Advertising increases revenue directly, but it also makes the environment more cognitively demanding. The platform, therefore, faces a trade-off: increasing advertising monetizes attention, yet excessive distortion discourages learning and reduces trade.

The core mechanism is simple. In markets where evaluating quality is inherently easy, consumers can assess the quality with little effort. There is limited monetizable attention, and transaction fees dominate as a source of revenue. As intrinsic complexity rises, consumers must devote more effort to learning. The scope for monetizing attention expands, and advertising becomes more attractive. However, once complexity is sufficiently high, further distortion reduces the set of consumers who find it worthwhile to learn. Trade declines, attention shrinks, and advertising loses effectiveness. Advertising intensity is therefore highest in markets of intermediate cognitive complexity.

Transaction fees respond differently. As markets become more complex and learning becomes fragile, participation becomes more sensitive to distortion. Optimal fees decline. Total profit decreases with complexity, yet the composition of profit shifts: the advertising share follows a hump-shaped pattern. Platforms that can use both instruments are structurally advantaged because they can substitute between extraction margins. Restricting the platform to a single instrument—either advertising-only or fee-only—reduces profitability, particularly in intermediate-complexity environments.

This paper extends the rational inattention framework by treating effective information costs as endogenous to platform design rather than as exogenous primitives. Moreover, it connects this framework to the industrial organization of platforms by jointly modeling transaction pricing and attention monetization. The result is a unified theory of how platforms optimally distort cognitive environments.

The implications are direct. Attention monetization is not uniformly harmful or uniformly dominant. Its profitability depends critically on the inherent complexity of the market. Markets characterized by moderate informational difficulty create the strongest incentives for distortion. By contrast, in very simple or very complex markets, fee extraction plays a larger role. The framework thus provides a structured way to think about why some platforms rely heavily on advertising while others rely primarily on transaction fees.

2 Related Literature

This paper studies how a platform shapes market outcomes by manipulating the *information-processing environment* faced by consumers. It connects three mechanisms that are typically studied separately: (i) the design of posterior beliefs, (ii) endogenous information acquisition under cognitive constraints, and (iii) platform pricing and design instruments. The contribution is to embed strategic platform design inside a rational inattention (RI) framework and to treat the effective marginal cost of attention as a platform choice variable jointly determined with transaction fees.

Designing demand through information A large literature studies how information design shapes demand elasticities and pricing outcomes. In monopoly environments, Roesler and Szentes (2017) characterize buyer-optimal signal structures and show that consumers may prefer imperfect information because it induces lower monopoly prices. The optimal signal generates unit-elastic demand and efficient trade. In competitive markets, Armstrong and Zhou (2022) analyze how consumer information affects price competition. Firm-optimal information amplifies perceived differentiation and relaxes competition, while consumer-optimal information dampens differentiation and intensifies it.

More generally, flexible segmentation and information design characterize feasible surplus outcomes. Elliott, Galeotti, Koh, and Li (2023) provide a complete characterization of achievable producer and consumer surplus pairs when a platform designs both matching patterns and information structures in markets with competing firms. In these frameworks, the designer directly chooses a signal structure subject to Bayes plausibility, and equilibrium pricing responds to the induced posterior distribution.

The present paper differs in that the platform does not directly choose posterior beliefs. Instead, it alters the environment in which consumers allocate attention. Posterior beliefs emerge endogenously from a rational inattention problem. Demand is shaped indirectly, through information-processing costs rather than through direct Bayesian persuasion.

Endogenous attention and rational inattention The rational inattention framework introduced by Sims (2003) models agents as optimally allocating scarce cognitive resources. In discrete-choice settings, Matějka and McKay (2015) show that RI yields multinomial logit choice probabilities, providing a tractable microfoundation for stochastic demand. Matějka and McKay (2012) analyze equilibrium with rationally inattentive consumers, showing how limited search and price dispersion arise from optimal attention allocation, while Matějka (2016) studies pricing by a rationally inattentive seller.

In these models, information costs are typically exogenous primitives. Here, the effective marginal cost of processing information is endogenized as a strategic platform instrument. Advertising load, interface design, or content congestion raise the cognitive cost of extracting payoff-relevant information, thereby altering the consumer's optimal signal choice. The information structure is not imposed; it results from optimal attention under platform-chosen costs.

The analysis is also related to work studying information provision when consumers possess outside information. Ennuschat (2025) analyzes targeted disclosure with partially informed consumers and shows how a platform manipulates posterior mean distributions to affect demand elasticities. In that setting, belief updating remains Bayesian and costless. By contrast, belief formation here is embedded inside an RI problem, so that demand responds to distortions in the consumer's optimization problem rather than to direct posterior design.

Platforms, pricing, and design instruments The industrial organization of platforms has largely focused on pricing in two-sided markets (e.g., Rochet and Tirole (2003, 2006); Armstrong (2006); Weyl (2010); Hagiu and Wright (2015)). These models analyze transaction fees and participation decisions while taking the information environment as given. A complementary strand emphasizes that platforms also manage information and matching. Jullien and Pavan (2019) study how information dispersion affects participation and pricing incentives, while Elliott et al. (2023) characterize welfare outcomes under joint matching and information design.

Other work highlights how firms exploit attention frictions and comparison costs. Ellison and Ellison (2009) formalize obfuscation incentives, and empirical evidence documents substantial heterogeneity in online search consistent with economically meaningful attention constraints (De Los Santos, Hortaçsu, & Wildenbeest, 2012). Recent research studies disclosure, steering, and shrouding as non-price instruments that shape effective competition (e.g., Johnen and Somogyi (2024)).

This paper complements these approaches by providing a disciplined information-theoretic mechanism through which advertising load increases the marginal cost of processing information for rationally inattentive consumers. The platform's problem is two-dimensional: it sets transaction fees and chooses the cognitive environment that governs attention allocation. These instruments are linked because demand elasticities are equilibrium objects determined by optimal information acquisition.

Contribution Existing work shows that posterior beliefs can be designed to manipulate demand and that attention constraints can generate endogenous stochastic choice. This paper combines these insights by allowing a platform to strategically manipulate the cost of attention itself. Relative to information-design models, direct posterior choice is replaced with an RI microfoundation. Relative to standard RI models, the information cost parameter is endogenized. Relative to platform IO models, fee extraction and attention monetization are linked through equilibrium demand elasticities.

In short, the paper identifies a distinct mechanism through which platforms shape market outcomes: by raising the marginal cost of processing information, they distort attention allocation, alter demand elasticities, and jointly determine transaction fees and advertising revenues.

3 Benchmark theoretical model

My model describes an online peer-to-peer platform where sellers can offer their items to buyers (the platform does not produce any items; it just runs the marketplace, where demand and supply meet). The primary focus of this model is to determine the platform's optimal information policy, given that users are rationally inattentive. Generally, the firm profits from

1. A fixed fee from every purchase.
2. The attention paid by the users. Here, I model it as the time spent on the website, where information is costly to acquire.

I assume the platform controls the proportion of the website covered by advertising. Advertising reduces effective processing capacity by shrinking usable interface space, thereby increasing marginal processing costs proportionally. As for the per-purchase fee, I model it as w, a fixed amount the user pays upon buying the good. The buyer controls how much information to acquire (in terms of mutual entropy) and whether to buy the item. The basic intuition is that high information cost forces buyers to spend more time on the website. However, because buyers are rationally inattentive, they may decide not to gather more information, which could reduce the likelihood of making a purchase.

3.1 Buyer's problem

Assume that there are two groups of sellers of equal size on the platform, denoted by $\omega = H$ and $\omega = L$. These groups differ in the quality of items they offer. The expected utility of the item from H seller, U_H, is equal to 1, while the expected utility of buying from L, U_L, is equal to 0. The buyer has an outside option that delivers utility of ε. Let there be a continuum of agents that differ in their outside option valuation, distributed according to $\varepsilon \sim \mathcal{U}\,(0{,}1)$[1]. All buyers, though, have the same correct flat prior belief about the seller's type[2]. The user encounters an item offered by an unknown seller and decides how much information to acquire and whether to purchase it. We model the buyer's decision-making process as a rational inattention problem - maximization of expected utility minus the cost of information

$$max_{Y\in\{0,1\},I}\mathbf{E}\left[u(Y{,}\omega)|I\right] - \lambda\frac{1}{(1-\alpha)}\cdot MI, \tag{1}$$

where Y represents the action (buy/not buy), I is the information acquired, and MI is the cost of that information (here, mutual information). Utility $u(Y,\omega)$ depends on the action taken and the true state of the world (H or L) and is given as follows

$$\begin{aligned} u(Y=1,\omega=H)=1-w, \quad u(Y=1,\omega=L)=-w,\\ u(Y=0,\omega=H)=u(Y=0,\omega=L)=\varepsilon, \end{aligned} \tag{2}$$

where w is the platform's fee.

3.2 Platform's problem

The platform derives profits from transaction fees and from advertising exposure generated by user attention. Let $w > 0$ denote the per-transaction fee and $\alpha \in (0{,}1)$ the fraction of the interface covered by advertisements. The complexity of the market, given purely by its nature, is exogenously set to λ. The higher λ, the more complicated it is to acquire, understand, and internalize information about the products. For instance, imagine a market for shoes and a market for laptops. When buying a laptop, one has to consider many technical parameters, whereas for shoes, the only criteria are appearance and size, which are easy to assess. Thus, λ captures inherent informational complexity, while α represents endogenous distortion of that environment.

Advertising increases the cognitive cost of processing information, so that the effective attention cost is

$$\tau = \frac{\lambda}{1-\alpha} \tag{3}$$

Since τ scales the marginal cost of processing one unit of information, total cognitive effort — and hence time spent — is proportional to $\tau \cdot MI$. The platform monetizes only the fraction α of this time covered by ads.

The platform, therefore, solves

[1] This assumption may be relaxed later

[2] The equal prior simplifies exposition; the qualitative structure survives asymmetric priors.

$$\max_{\alpha,w} \Pi_F(\alpha,w) + \Pi_I(\alpha,w) \tag{4}$$

where transaction profits are

$$\Pi_F(\alpha,w) = w \int_0^1 P^*(Y = 1|\alpha,w)\, d\varepsilon \tag{5}$$

and advertising profits are

$$\Pi_I(\alpha,w) = \frac{\lambda\alpha}{1-\alpha} \int_0^1 MI^*(\alpha,w)\, d\varepsilon \tag{6}$$

where starred objects denote the buyer's optimal behavior.

4 Results

4.1 Buyer's problem

We begin by solving the buyer's problem. A user encounters an item offered by a seller whose type $\omega \in \{H,L\}$ is unknown, with prior probability $1/2$.

The buyer chooses how much information to acquire about seller quality before deciding whether to buy ($Y = 1$) or not ($Y = 0$). The decision problem follows the rational inattention framework and can be written as:

$$\max_{P(Y|\omega)} \sum_{\omega\in\{H,L\}} \frac{1}{2} \sum_{Y\in\{0,1\}} P(Y|\omega)\, u(Y,\omega) - \tau \cdot MI(Y;\omega). \tag{7}$$

Define the effective cost of attention as

$$\tau = \frac{\lambda}{1-\alpha} \tag{8}$$

The buyer's utility is

$$u(1,H) = 1 - w \tag{9}$$

$$u(1,L) = -w \tag{10}$$

$$u(0,H) = u(0,L) = \varepsilon \tag{11}$$

Mutual information is given by

$$MI = \sum_{Y\in\{0,1\}} \sum_{\omega\in\{H,L\}} \frac{1}{2} P(Y|\omega) \log\left(\frac{P(Y|\omega)}{P(Y)}\right) \tag{12}$$

Following standard results, the optimal choice probabilities satisfy a logit form:

$$P^*(Y=1|\omega) = \frac{1}{1 + \frac{P^*(Y=0)}{P^*(Y=1)} \exp\left(\frac{u(0,\omega)-u(1,\omega)}{\tau}\right)} \tag{13}$$

Using the utility structure, this becomes

$$P^*(1|H) = \frac{1}{1 + k\exp\left(\frac{\varepsilon-1+w}{\tau}\right)} \tag{14}$$

$$P^*(1|L) = \frac{1}{1 + k\exp\left(\frac{\varepsilon+w}{\tau}\right)} \tag{15}$$

where

$$k = \frac{P^*(Y=0)}{P^*(Y=1)} = \frac{P^*(0)}{P^*(1)} = \frac{1-P^*(1)}{P^*(1)}. \tag{16}$$

The implied unconditional purchase probability is

$$P^*(1) = \frac{1}{2}\left(\frac{1}{1+ke^{\frac{\varepsilon-1+w}{\tau}}} + \frac{1}{1+ke^{\frac{\varepsilon+w}{\tau}}}\right).$$

Consistency requires

$$P^*(1) = \frac{1}{1+k}.$$

Equating the two expressions and multiplying both sides by $2(1+k)\left(1+ke^{\frac{\varepsilon-1+w}{\tau}}\right)\left(1+ke^{\frac{\varepsilon+w}{\tau}}\right)$ yields a linear equation in k, whose interior solution is

$$k^* = -\frac{\left(1+e^{-1/\tau}\right)e^{\frac{\varepsilon+w}{\tau}} - 2}{2e^{-1/\tau}e^{\frac{2(\varepsilon+w)}{\tau}} - \left(1+e^{-1/\tau}\right)e^{\frac{\varepsilon+w}{\tau}}}.$$

This solution is valid only when it satisfies $k^* > 0$, which defines the interior region with thresholds

$$\varepsilon_0 = \tau\log\left(\frac{2}{1+e^{-1/\tau}}\right) - w \tag{17}$$

$$\varepsilon_1 = -\tau\log\left(\frac{2}{1+e^{1/\tau}}\right) - w \tag{18}$$

Note that $\varepsilon_0 < \varepsilon_1$ for all $\tau > 0$, so the interior region is well-defined. Then:

- If $\varepsilon < \varepsilon_0$, the buyer always purchases:

$$P^*(1|H) = P^*(1|L) = 1$$

- If $\varepsilon > \varepsilon_1$, the buyer never purchases:

$$P^*(1|H) = P^*(1|L) = 0$$

- If $\varepsilon \in [\varepsilon_0, \varepsilon_1]$, the interior solution above applies.

Figure 1 shows the best response of the user as a function of ε for given w and λ. Panel (a) depicts the unconditional buying probability, which is weakly decreasing in ε, the outside option valuation. There is a region of always-buyers with low outside option valuation. The interior region consists of buyers who purchase with some nontrivial probability. Finally, agents with high outside-option valuation never buy anything.

Panel (b) shows the buyer-optimal amount of information acquired (MI). The always-buyers and never-buyers do not acquire any information. In the interior region, agents invest in information acquisition. The function follows an inverse-U shape, resembling that of agents whose outside option is close to the expected utility of buying; they are the ones who try to learn the most.

4.2 Platform's Problem

We now turn to the platform's profit. The platform knows the ε distribution and buyer's objectives. Given that, it can fine-tune w and α to maximize the available profit.

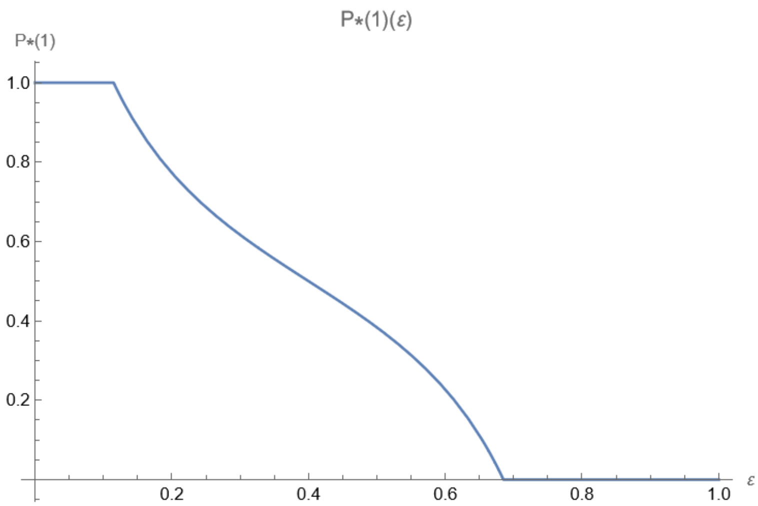
Again, we will need to distinguish three cases based on whether $\varepsilon = 0$ lies below, inside, or above the interior region $[\varepsilon_0, \varepsilon_1]$ (equivalently, whether B_w lies below, inside, or above $[B_0, B_1]$).

- **Case 1: Full interior region** If $\varepsilon_0 \geq 0$, there are some always-buyers and the whole region of nontrivial probability buyers. Transaction profit simplifies to

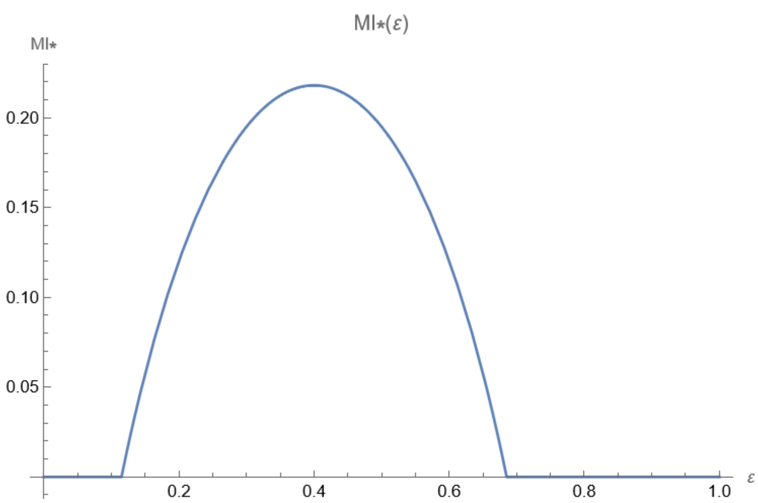
$$\Pi_F = w\left(\frac{1}{2} - w\right) \tag{19}$$

  Advertising profit is

$$\Pi_I = \frac{\lambda\alpha}{(1-\alpha)} \int_{\varepsilon_0}^{\varepsilon_1} MI(\varepsilon)\, d\varepsilon \tag{20}$$



(a) Unconditional buying probability



(b) Information acquired (MI)

Figure 1: Buyer's problem solution as a function of the outside option valuation ε for $\lambda = 0.2$, $w = 0.1$, $\alpha = 0.4$

- **Case 2: Partial interior region** If $\varepsilon_0 \leq 0$, the transaction profit is

$$\Pi_F = \frac{w}{2} \log \left(\frac{\frac{(1-\frac{\alpha-1}{\lambda})^2}{4\frac{\alpha-1}{\lambda}}}{\left(\exp\left(\frac{(1-\alpha)w}{\lambda}\right) - 1\right)\left(\exp\left(\frac{\alpha-1+(1-\alpha)w}{\lambda}\right) - 1\right)} \right) \tag{21}$$

and

$$\Pi_I = \frac{\lambda\alpha}{(1-\alpha)} \int_0^{\varepsilon_1} MI(\varepsilon) d\varepsilon \tag{22}$$

- **Case 3: No interior region** If $\varepsilon_1 \leq 0$

$$\Pi_I = \Pi_F = 0 \tag{23}$$

Figure 3 shows the expected utility for different agents across the outside option valuation scale. For comparison, it also shows expected utility under two benchmark scenarios. First, in the absence of the market, all agents would be left with their outside options. Second, I plot the expected utility under the no-fee, no-advertising scenario as the ideal for buyers. The equilibrium utility does not attain this maximum. The introduction of the market increases the expected utility for agents with lower outside-option valuations. The benefit diminishes with growing ε, nevertheless, no agent is worse-off in expectation compared to the *no market* scenario. User welfare is weakly decreasing in λ, the environment's complexity.

4.3 Comparative statics

In this section, I plot the relevant outcomes as a function of λ. For each λ, the platform chooses (α,w) anticipating buyers' optimal RI response; we refer to the resulting outcome as the (platform-led) equilibrium.

Figure 4 illustrates equilibrium outcomes as a function of λ, the overall complexity of the choice environment. Recall that λ scales the cognitive cost of processing information, while advertising further amplifies this cost through $\tau = \lambda/(1-\alpha)$. Hence larger λ corresponds to a more cognitively demanding market. Increasing λ raises baseline processing cost, which (i) reduces optimal information acquisition (MI^*), (ii) shrinks the set of learning buyers, (iii) lowers purchase probability, and (iv) changes the platform's optimal mix between w and α

Panel (a) shows the equilibrium advertising load α^*. The relationship between complexity and optimal advertising is non-monotonic. When λ is small, the decision problem is simple. Users can easily identify high-quality sellers, and the platform has little incentive to increase advertising

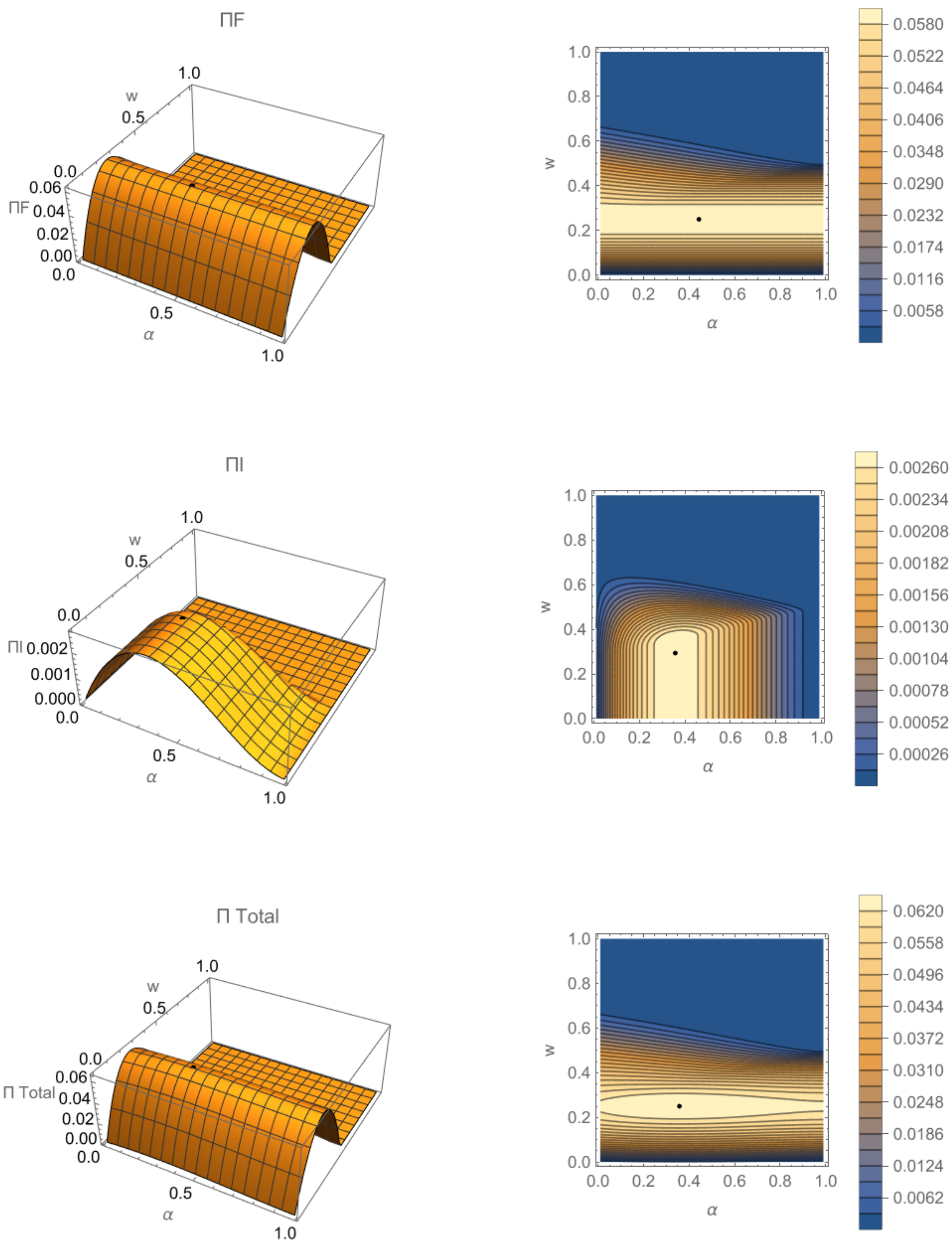


Figure 2: Platform profit and its decomposition into advertising profit Π_I and purchase profit Π_F. The black dot indicates the equilibrium; $\lambda = 0.2$

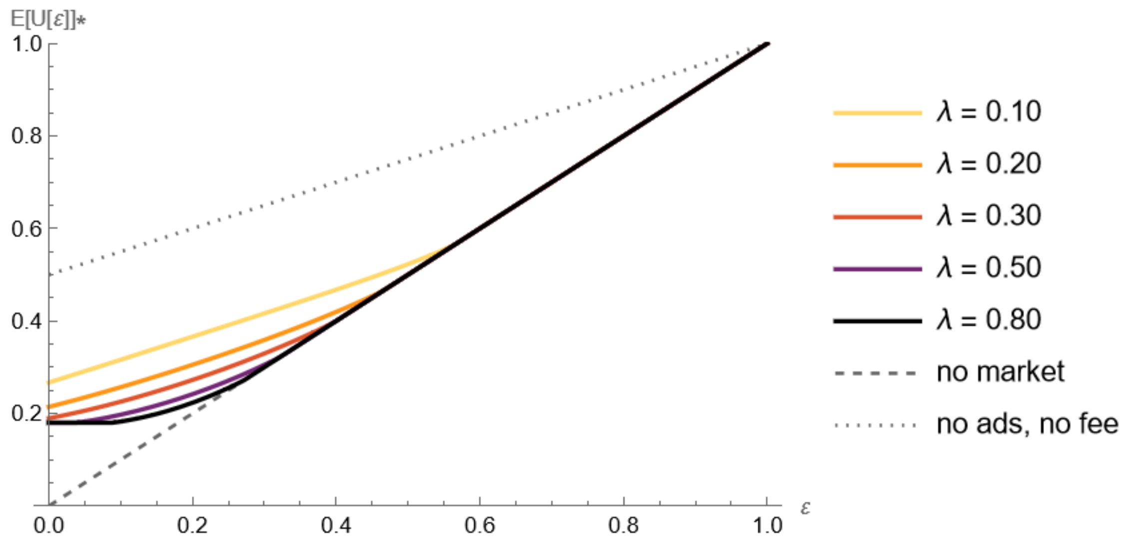
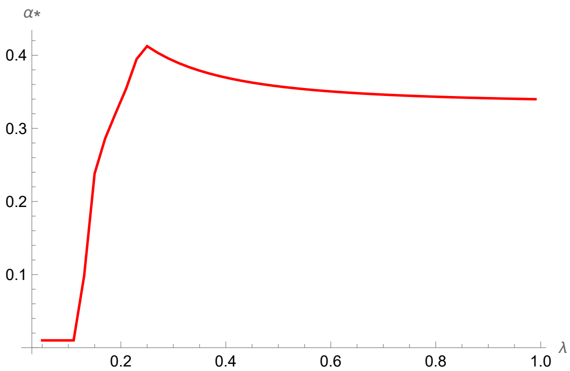


Figure 3: Equilibrium expected utility for buyers with different outside option valuations ε (red solid line). The dashed line shows the expected utility absent the market, when everyone is left with their outside option. The dotted line shows the user profit if there were no fees and no advertising ($w = 0$, $\alpha = 0$). Different colors represent different values of λ.

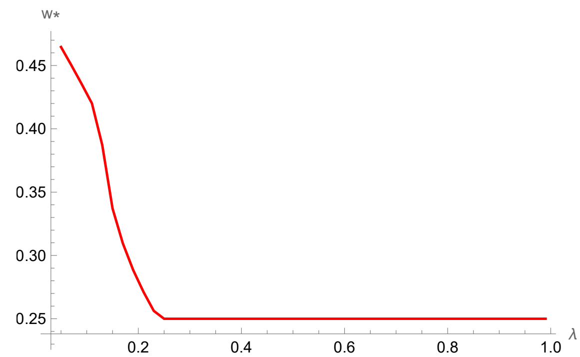
intensity. Distorting the information environment generates little additional attention and only reduces transaction surplus. Consequently, optimal advertising is close to zero. As λ increases, the environment becomes more cognitively demanding. Users must devote more effort to distinguish high- and low-quality sellers. Advertising now becomes a powerful instrument: by increasing the effective attention cost, the platform increases time spent on the website, which directly raises advertising revenue. Optimal advertising, therefore, increases sharply. However, when λ becomes sufficiently large, additional advertising begins to shrink the interior region of users who optimally process information. Excessive distortion reduces the market's informativeness, dampens purchase incentives, and ultimately lowers both transaction and advertising revenues. Accordingly, optimal advertising intensity declines and stabilizes. Thus, advertising is most aggressive at intermediate levels of complexity.

Panel (b) shows the equilibrium fee w^*. When λ is small, users can reliably identify high-quality sellers. The platform can therefore extract a relatively high fee. As complexity increases, user uncertainty rises, and participation becomes more fragile. To maintain trading activity, the platform reduces the transaction fee. For sufficiently large λ, the fee converges to $w = 1/4$. Hence, increasing complexity shifts the platform away from transaction-based extraction.

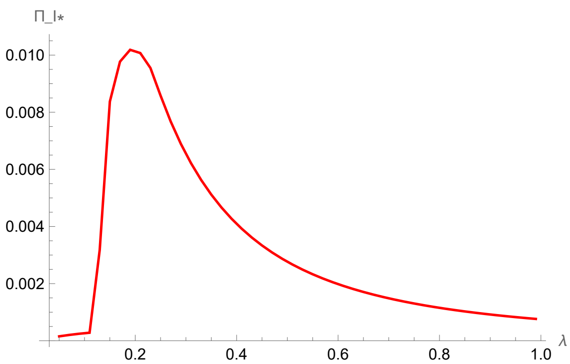
Panels (c) and (d) display advertising and fee profits. Advertising revenue is hump-shaped in λ. Recall that the interior region consists of types ε for which $MI^*(\varepsilon) > 0$. At low complexity, there is little attention to monetize. At an intermediate level of complexity, users must devote substantial cognitive effort to maximize monetizable attention. At high complexity, however, the



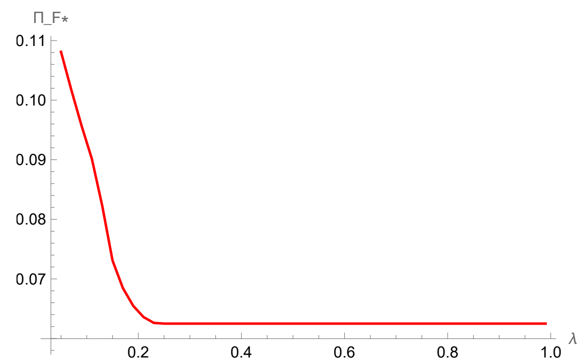
(a) α^* - equilibrium advertising load



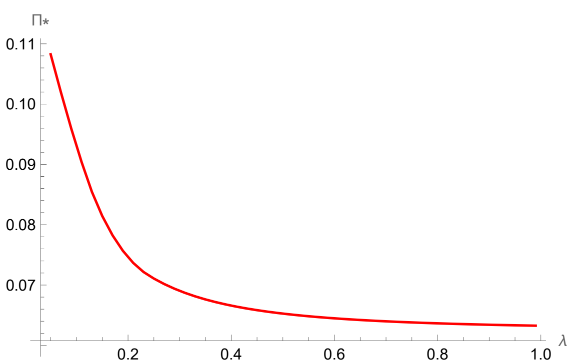
(b) w^* - equilibrium purchase fee



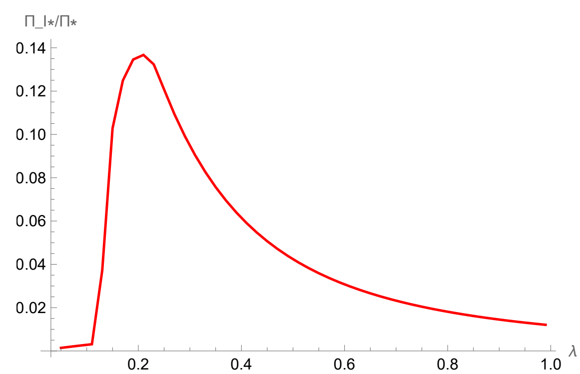
(c) Π_I^* - equilibrium advertising profit



(d) Π_F^* - equilibrium fee profit



(e) Π^* - equilibrium total profit



(f) Π_I^*/Π^* - equilibrium advertising profit share

Figure 4: Equilibrium outcomes as a function of λ - the overall complexity of the choice problem

interior region collapses and mutual information declines, reducing advertising revenue. In contrast, fee revenue declines monotonically with complexity. As users become less confident in their assessments, the platform's ability to extract surplus through transaction fees diminishes.

Panel (e) shows total profit, which decreases in λ. Although the platform optimally adjusts both instruments, it cannot fully offset the reduction in trade surplus caused by higher baseline processing costs.

Panel (f) shows the share of advertising revenue in total profit. The advertising share is hump-shaped. For simple markets, the platform behaves like a classical transaction intermediary. For intermediate levels of complexity, advertising becomes relatively more important (peaking at 14% share of total profit), though transaction fees remain the primary revenue source in this calibration.

To sum up, Figure 4 (where the optimal policies were determined numerically) suggests the following stylized facts in the baseline calibration

1. $\alpha^*(\lambda)$ is single-peaked
2. $w^*(\lambda)$ weakly decreasing with limit $w^* = 1/4$
3. $\Pi_I^*(\lambda)$ single-peaked, $\Pi_F^*(\lambda)$ decreasing
4. $\Pi_I^*(\lambda)/\Pi^*(\lambda)$ single-peaked and bounded above (e.g., 14 % in this calibration)

4.4 Path towards closed-form policy functions

To obtain analytical platform policies, I introduce two main approximations that reduce the platform problem to low-degree polynomial conditions; in Case 1, the FOC is a cubic in α.

Approximation 1 (Parabolic mutual information in ε). Within the interior region $[\varepsilon_0,\varepsilon_1]$, mutual information is approximated by a quadratic function that matches the two zero endpoints and the peak at the midpoint:

$$MI(\varepsilon) \;\approx\; -\frac{4\,M(\tau)}{(\varepsilon_1-\varepsilon_0)^2}(\varepsilon-\varepsilon_0)(\varepsilon-\varepsilon_1),$$

where $M(\tau) := \max_{\varepsilon\in[\varepsilon_0,\varepsilon_1]} MI(\varepsilon)$ denotes peak mutual information, and

$$\Delta(\tau) \equiv \varepsilon_1 - \varepsilon_0$$

is the width of the interior region. This implies the area formula

$$\int_{\varepsilon_0}^{\varepsilon_1} MI(\varepsilon)\,d\varepsilon = \frac{2}{3}M(\tau)\Delta(\tau).$$

Approximation 2 (Linearization in ad intensity). Let $\tau = \lambda/(1-\alpha)$. We approximate both peak information and interior width by linear functions of advertising intensity around $\alpha = 0$:

$$M(\alpha) \approx M_0(\lambda) + M_1(\lambda)\alpha, \qquad \Delta(\alpha) \approx \Delta_0(\lambda) + \Delta_1(\lambda)\alpha,$$

where coefficients are functions of λ and are reported in the Appendix.

For and only for the derivation of the Case 2 Transaction profit, an additional approximation is needed. **Approximation 3 (local polynomial approximation in w).** To obtain algebraic first-order conditions, approximate $\Pi_F^{(2)}(\alpha,w)$ by a cubic Taylor polynomial in w around $w_0 = \frac{1}{4}$:

$$\Pi_F^{(2)}(\alpha,w) \approx F_0(\alpha;\lambda) + F_1(\alpha;\lambda)(w-w_0) + \frac{1}{2}F_2(\alpha;\lambda)(w-w_0)^2 + \frac{1}{6}F_3(\alpha;\lambda)(w-w_0)^3,$$

where

$$F_k(\alpha;\lambda) = \frac{\partial^k}{\partial w^k}\Pi_F^{(2)}(\alpha,w)\bigg|_{w=w_0}, \qquad k = 0,1,2,3, \qquad w_0 = \tfrac{1}{4}.$$

Under these approximations, advertising profit in Case 1 (full interior region) becomes

$$\Pi_I^{(1)}(\alpha;\lambda) = \frac{2}{3}\frac{\lambda\alpha}{1-\alpha}\big(M_0 + M_1\alpha\big)\big(\Delta_0 + \Delta_1\alpha\big).$$

Transaction profit remains

$$\Pi_F(w) = w\left(\frac{1}{2} - w\right),$$

implying

$$w^{*(1)} = \frac{1}{4}, \qquad \Pi_F^{*(1)} = \frac{1}{16}.$$

Case 1 equilibrium policy.

Let

$$A_0(\lambda) = M_0(\lambda)\Delta_0(\lambda), \qquad A_1(\lambda) = M_0\Delta_1 + M_1\Delta_0, \qquad A_2(\lambda) = M_1\Delta_1.$$

The optimal advertising intensity in Case 1, $\alpha^{*(1)}(\lambda)$, is the unique root in $(0,1)$ of the cubic polynomial

$$A_2\,\alpha^3 + (2A_1 - A_2)\alpha^2 + (2A_0 - A_1)\alpha + A_0 = 0.$$

Since cubic equations admit closed-form solutions, $\alpha^{*(1)}(\lambda)$ is available in closed form. Equilibrium profit is therefore

$$\Pi^{*(1)}(\lambda) = \frac{1}{16} + \frac{2}{3}\frac{\lambda\alpha^{*(1)}(\lambda)}{1-\alpha^{*(1)}(\lambda)}\big(M_0 + M_1\alpha^{*(1)}(\lambda)\big)\big(\Delta_0 + \Delta_1\alpha^{*(1)}(\lambda)\big).$$

Case 2 equilibrium policy.

In Case 2 (partial interior region), the ad-profit term becomes

$$\Pi_I^{(2)}(\alpha,w;\lambda) = \frac{\lambda\alpha}{1-\alpha}\big(M_0 + M_1\alpha\big) f\big(\Delta_0 + \Delta_1\alpha,\, x_a(\alpha,w)\big),$$

where

$$f(\Delta,x) = \frac{2}{3}\Delta - \frac{2x^2}{\Delta} + \frac{4x^3}{3\Delta^2}$$

and $x_a(\alpha,w)$ is the (linearized) lower cutoff of the interior region.

The resulting FOCs are rational polynomials.

Result. Under the parabolic approximation of mutual information and linearization in advertising intensity, equilibrium platform policies exist in closed form. Case 1 yields a cubic in α and thus an explicit solution. Case 2 yields a polynomial system admitting closed-form solutions. All coefficients depend only on λ.

5 Extensions

Although the baseline model describes a retail platform, it also applies to other business models. We consider two important special cases: (i) media platforms that monetize only attention and cannot charge transaction fees, and (ii) physical marketplaces that monetize only transactions and cannot monetize attention.

5.1 Media Platforms (Ad-Only)

Media platforms (e.g. online news, social networks) monetize solely through advertising. Thus $w = 0$ and $\Pi_F \equiv 0$. The platform therefore always operates with the full interior region of buyers (since $w = 0$ implies $B_w = 1$ and $B_0 > 1$ for all τ, the economy is in Case 1).

Under the parabolic approximation of mutual information in ε and a first-order linear approximation in advertising intensity,

$$M(\alpha) \approx M_0(\lambda) + M_1(\lambda)\alpha, \qquad \Delta(\alpha) \approx \Delta_0(\lambda) + \Delta_1(\lambda)\alpha,$$

advertising profit becomes

$$\Pi^{\text{med}}(\alpha;\lambda) = \frac{2}{3}\frac{\lambda\alpha}{1-\alpha}\big(M_0 + M_1\alpha\big)\big(\Delta_0 + \Delta_1\alpha\big).$$

Define

$$A_0 = M_0\Delta_0, \quad A_1 = M_0\Delta_1 + M_1\Delta_0, \quad A_2 = M_1\Delta_1.$$

The optimal advertising intensity is the unique root in $(0,1)$ of

$$A_2\alpha^3 + (2A_1 - A_2)\alpha^2 + (2A_0 - A_1)\alpha + A_0 = 0.$$

Since cubic equations admit closed-form solutions, $\alpha^*_{\text{med}}(\lambda)$ exists in closed form. Equilibrium profit equals

$$\Pi^*_{\text{med}}(\lambda) = \frac{2}{3}\frac{\lambda\alpha^*_{\text{med}}(\lambda)}{1-\alpha^*_{\text{med}}(\lambda)}\big(M_0 + M_1\alpha^*_{\text{med}}(\lambda)\big)\big(\Delta_0 + \Delta_1\alpha^*_{\text{med}}(\lambda)\big).$$

5.2 Physical Marketplaces (Fee-Only)

In physical marketplaces, attention cannot be monetized. Hence $\alpha = 0$ and $\Pi_I \equiv 0$. The platform chooses only w.

When the interior region is nonempty, transaction profit simplifies to

$$\Pi_F(w) = w\Big(\frac{1}{2} - w\Big),$$

with unconstrained maximizer $w^\circ = \frac{1}{4}$.

However, the interior region exists only if

$$\varepsilon_0 \geq 0 \quad \Longleftrightarrow \quad w \leq \bar{w}(\lambda),$$

where

$$\bar{w}(\lambda) = \lambda \log\Big(\frac{2}{1+e^{-1/\lambda}}\Big).$$

Define the threshold $\lambda^\dagger$ as the unique solution to

$$\lambda^\dagger \log\Big(\frac{2}{1+e^{-1/\lambda^\dagger}}\Big) = \frac{1}{4}.$$

Then the equilibrium fee policy is

$$\boxed{w^*_{\text{phys}}(\lambda) = \begin{cases} \frac{1}{4}, & \lambda \geq \lambda^\dagger, \\ \lambda \log\Big(\frac{2}{1+e^{-1/\lambda}}\Big), & \lambda < \lambda^\dagger, \end{cases} \qquad \alpha^*_{\text{phys}} = 0.}$$

Equilibrium profit equals

$$\Pi^*_{\text{phys}}(\lambda) = w^*_{\text{phys}}(\lambda)\Big(\frac{1}{2} - w^*_{\text{phys}}(\lambda)\Big).$$

Numerically,

$$\boxed{\lambda^\dagger \approx 0.463.}$$

Thus, when information costs are sufficiently high ($\lambda \geq 0.463$), the physical marketplace sets the interior optimum fee $w = 1/4$. When information costs are low, the fee is constrained by the boundary of the interior region.

5.3 Comparison

Figure 5 compares expected buyer utility across the three business models as a function of the outside option ε. The dashed line represents the no-market benchmark, while the dotted line corresponds to the frictionless case with no fees and no advertising ($w = 0$, $\alpha = 0$).

All equilibrium utility curves lie above the no-market benchmark for sufficiently low ε, reflecting gains from trade, and converge to the outside option as ε increases. The frictionless benchmark dominates equilibrium allocations because platform monetization distorts the information environment, the transaction margin, or both.

Across models, the media platform delivers higher utility to low-ε buyers. The physical marketplace distorts utility through fee extraction, discouraging low-ε buyers from engaging in the market and lowering their expected utility. The baseline platform lies between these cases. The comparison highlights that in the case of media, the user welfare is weakly larger in expectation than the benchmark and physical media case.

Figure 6 compares equilibrium outcomes across three business models as a function of λ, the overall complexity of the choice environment: the baseline retail platform (red), an ad-only media platform (blue), and a fee-only physical marketplace (green). Since λ scales the cognitive cost of processing information, a higher λ reduces optimal information acquisition, shrinks the interior region of learning buyers, and weakens trade.

Panel (a) plots equilibrium advertising intensity α^*. The media platform relies exclusively on advertising and therefore chooses a higher α^* overall, though it too reduces ad load as complexity becomes excessive. The physical marketplace sets $\alpha^{\equiv}0$ by construction.

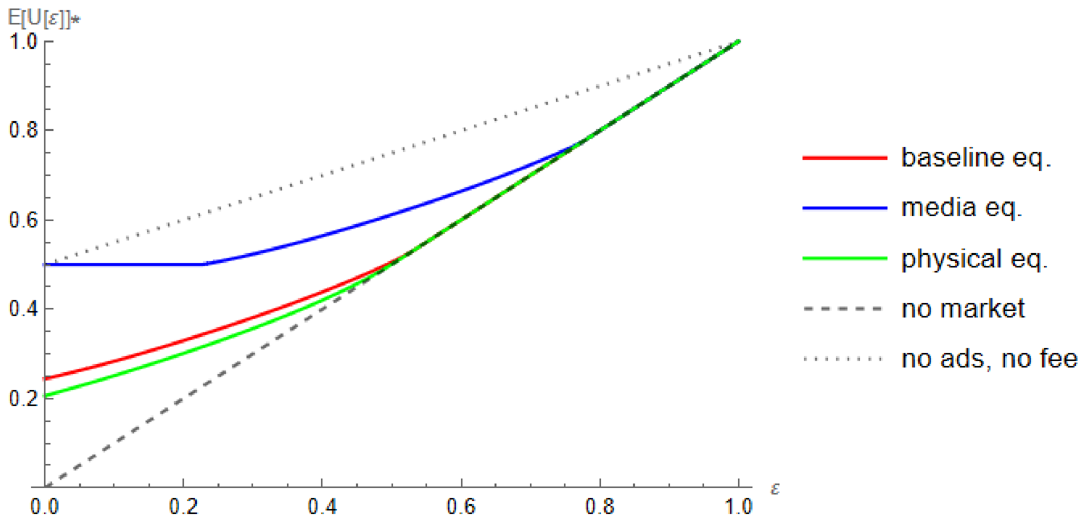
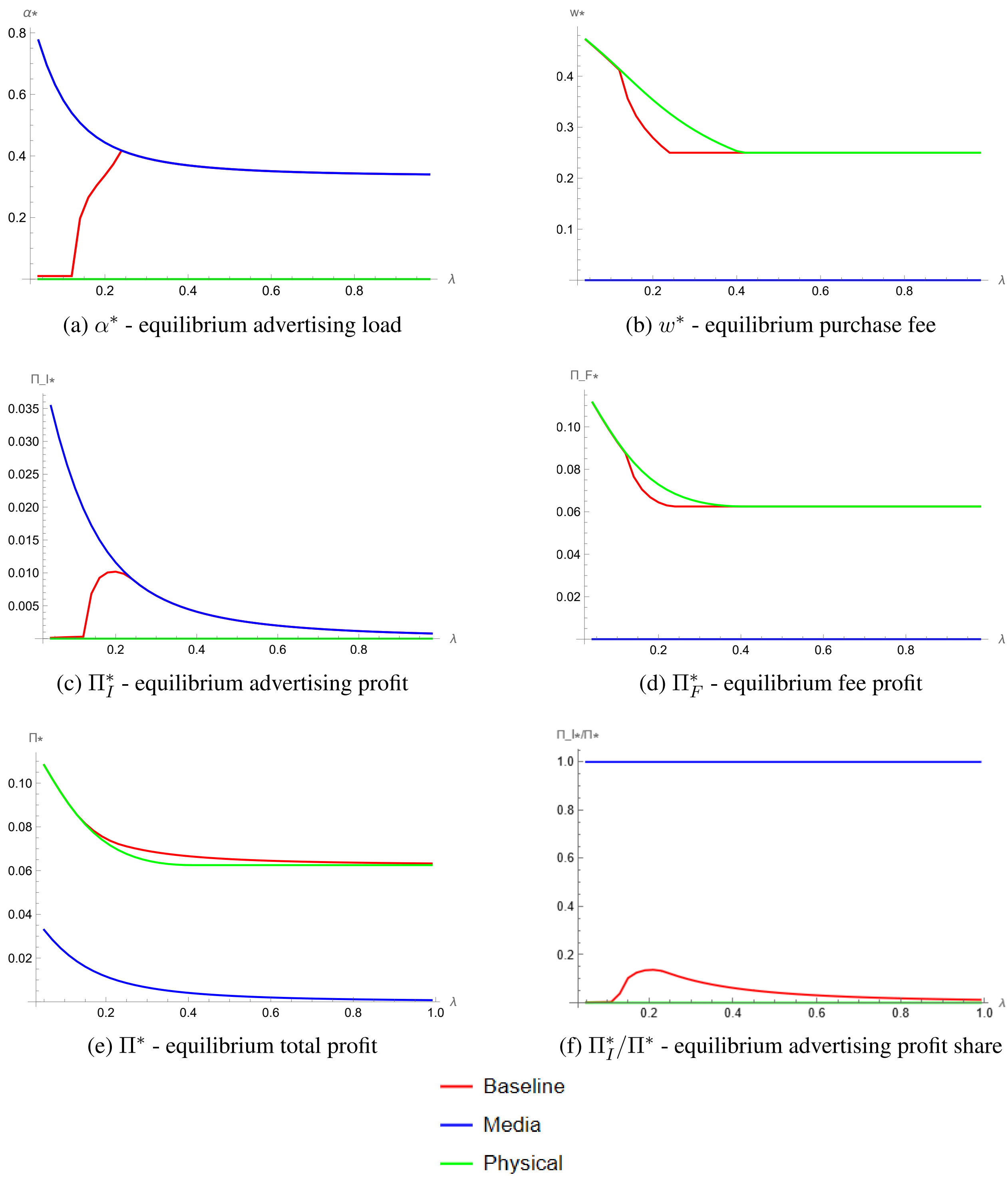


Figure 5: Equilibrium expected utility for buyers with different outside option valuations ε (red solid line). The dashed line shows the expected utility absent the market, when everyone is left with their outside option. The dotted line shows the user profit if there was no fee and no advertising ($w = 0$, $\alpha = 0$); $\lambda = 0.2$

Panel (b) shows the equilibrium transaction fee w^*. The physical marketplace seems to be similar to the benchmark, but at intermediate λ the optimal fee is higher since there is no opportunity for attention profit. Once λ is sufficiently large, the physical marketplace fee converges to $w^* = 1/4$. In the media model, $w \equiv 0$.

Panels (c) and (d) decompose profits. Advertising profit Π_I^* is positive only when $\alpha > 0$. In the media model, advertising profit is highest at low complexity and declines monotonically as processing becomes too costly. Fee profit Π_F^* for physical marketplace declines with λ, reflecting weaker participation and lower effective surplus extraction.

Panel (e) shows total profit, which decreases in λ across all models. The baseline platform attains the highest profit because it can substitute between instruments. The media model performs significantly worse than the physical marketplace.



(a) α^* - equilibrium advertising load

(b) w^* - equilibrium purchase fee

(c) Π_I^* - equilibrium advertising profit

(d) Π_F^* - equilibrium fee profit

(e) Π^* - equilibrium total profit

(f) Π_I^*/Π^* - equilibrium advertising profit share

Figure 6: Equilibrium outcomes as a function of λ - the overall complexity of the choice problem

6 Discussion

Advertising–fee trade-off The benchmark model highlights a structural trade-off between transaction-based extraction and attention-based monetization. The platform controls two instruments: the per-purchase fee w and the advertising load α, which increases the effective cost of attention $\tau = \lambda/(1-\alpha)$. These instruments affect different margins.

An increase in w directly extracts surplus from inframarginal buyers but compresses the set of participating and learning agents. An increase in α raises the cost of information processing. Because advertising revenue scales with $\tau \cdot MI$, moderate increases in α can raise monetizable attention. However, excessive distortion shrinks the interior region of learning buyers and reduces both mutual information and trade.

The comparative statics with respect to λ clarify the mechanism. When λ is low, the decision problem is simple. Buyers can easily distinguish seller types, the interior region is small, and monetizable attention is limited. The platform relies primarily on transaction fees. As λ increases, buyers devote more cognitive effort to learning, expanding monetizable attention. Advertising becomes valuable, and $\alpha^{(}\lambda)$ rises. When λ becomes sufficiently large, however, cognitive frictions dominate. The interior region contracts and mutual information declines. At that point, further distortion reduces total surplus, and optimal advertising decreases. Accordingly, $\alpha^{(}\lambda)$ is single-peaked, $w^*(\lambda)$ declines and converges to the interior benchmark, and total profit decreases in λ.

The platform performs best in environments of intermediate complexity, where attention is valuable but not prohibitively costly.

Media and physical marketplaces Restricting the platform to a single revenue instrument alters equilibrium policies and profit composition. An ad-only media platform sets $w = 0$ and relies entirely on advertising. Advertising profit is highest when λ is low, and declines monotonically as cognitive processing becomes more costly. The model, therefore, predicts that ad-only platforms are particularly vulnerable in highly complex environments.

A fee-only physical marketplace sets $\alpha = 0$ and extracts surplus solely through transaction fees. Again, its total profit declines with complexity because attention cannot be monetized, but it still delivers much higher profit than the media business model.

The comparison suggests that multi-instrument (baseline) platforms are structurally advantaged in markets of intermediate cognitive complexity compared to fee-only (physical) businesses.

Welfare From the buyer's perspective, the frictionless benchmark ($w = 0$, $\alpha = 0$) is first-best. Any monetization reduces expected utility relative to this ideal. Relative to the no-market benchmark, however, all equilibria generate gains from trade for sufficiently low outside-option types. The equilibrium expected utility is weakly decreasing in λ for every agent.

The platform's profit is maximized at an intermediate level of complexity, while consumer welfare is highest in low-complexity environments with minimal monetization. The divergence reflects that the platform internalizes revenue but not the full welfare cost of attention distortion.

Illustrative Evidence: eBay Fee Structure Even though rigorous empirical verification would require detailed data and a careful econometric strategy, which fall outside the scope of this paper. Variation in eBay's fee schedule (eBay, 2026) across categories provides suggestive support for the model's predictions regarding w^* and cognitive complexity.

Categories plausibly associated with higher cognitive complexity or specialized decision-making — such as heavy equipment (3%), musical instruments (6.7%), or NFTs (5%) — incur relatively low final-value fees. In contrast, categories likely associated with simpler purchase decisions — books, media, clothing, and jewelry — face fees around 15%.

Additionally, eBay Inc. (2025) reports that third-party advertising makes up only up to 3 % of total revenues, which, again, is in line with my model.

Interpreting λ as intrinsic decision complexity, the model predicts that higher λ constrains fee extraction. The observed fee variation is broadly consistent with this prediction: high-complexity categories exhibit lower ad valorem fees and often declining marginal rates for large transactions. While descriptive, these patterns align with comparative statics: $w^*(\lambda)$ declines as cognitive complexity increases.

Policy implications Recent regulatory initiatives (*Digital Markets, Competition and Consumers Act 2024*, 2024, such as) focus on gatekeeper power, self-preferencing, and unfair trading conditions. A central concern is that dominant platforms may exploit users through opaque pricing structures or manipulative interface design.

My model provides a framework for interpreting such concerns. Advertising intensity α distorts the information environment by raising effective attention costs. From a welfare perspective, this resembles a non-price instrument that extracts surplus indirectly. In highly complex markets (high λ), the platform may rely more heavily on distortionary monetization, even though cognitive frictions already limit trade.

Regulatory policies that restrict advertising load, mandate transparency, or limit discriminatory fee structures effectively constrain the platform's choice of α and w. The model suggests that the welfare impact of such interventions depends critically on market complexity. In low-complexity markets, limiting advertising may increase welfare with minimal efficiency loss. In intermediate-complexity environments, however, restricting one instrument may push the platform to rely more heavily on the other.

More broadly, the analysis implies that the regulation of digital platforms should account for users' cognitive environments. Markets characterized by inherently complex decision problems may require stronger safeguards against attention-based distortion, whereas in simple markets, fee-based extraction may be the dominant margin of concern.

7 Conclusion

This paper studies platform pricing when users are rationally inattentive, and attention can be monetized. By allowing the platform to choose both transaction fees and advertising intensity, the model links direct surplus extraction with distortion of the information environment. Advertising increases the effective cost of processing information and therefore shapes both attention allocation and trade.

Three main insights emerge. First, optimal advertising intensity is single-peaked in intrinsic market complexity: it is low in simple markets, highest at intermediate complexity, and declines when excessive distortion undermines learning and trade. Second, transaction fees decline with complexity. Third, total profit decreases as complexity rises, while the advertising share of revenue follows a hump-shaped pattern. Platforms combining both instruments are structurally advantaged relative to ad-only or fee-only business models because they can substitute between extraction margins.

The analysis highlights that the profitability and welfare consequences of attention monetization depend critically on users' cognitive environments. Markets characterized by moderate informational complexity create the strongest incentives for distortion. By endogenizing attention costs within a rational inattention framework, the paper provides a tractable structure for studying how digital platforms shape, and profit from, the informational conditions under which consumers make decisions.

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Appendix

A Closed-form solution under linear approximation

This Appendix derives the equilibrium policies under two approximations:
(i) A quadratic (parabolic) approximation of mutual information in ε;
(ii) A first-order (linear) approximation in advertising intensity α.
Throughout, define

$$\tau(\alpha) = \frac{\lambda}{1-\alpha}.$$

Parabolic approximation of mutual information

Within the interior region $[\varepsilon_0, \varepsilon_1]$, mutual information is approximated by

$$MI(\varepsilon) \approx -\frac{4\,M(\tau)}{(\varepsilon_1 - \varepsilon_0)^2}(\varepsilon - \varepsilon_0)(\varepsilon - \varepsilon_1),$$

where $M(\tau)$ denotes peak mutual information and

$$\Delta(\tau) = \varepsilon_1 - \varepsilon_0$$

is the width of the interior region.
Integrating yields the exact area formula

$$\int_{\varepsilon_0}^{\varepsilon_1} MI(\varepsilon)\, d\varepsilon = \frac{2}{3} M(\tau)\Delta(\tau).$$

If the lower bound of the integral is $\varepsilon_a \in [\varepsilon_0, \varepsilon_1]$, define

$$x_a = \varepsilon_a - \varepsilon_0 \in [0, \Delta].$$

Then

$$\int_{\varepsilon_a}^{\varepsilon_1} MI(\varepsilon)\, d\varepsilon = M(\tau)\, f(\Delta, x_a),$$

where

$$f(\Delta, x) = \frac{2}{3}\Delta - \frac{2x^2}{\Delta} + \frac{4x^3}{3\Delta^2}.$$

Linear approximation in advertising intensity

We linearize both peak information and interior width around $\alpha = 0$:

$$M(\alpha) \approx M_0(\lambda) + M_1(\lambda)\alpha,$$

$$\Delta(\alpha) \approx \Delta_0(\lambda) + \Delta_1(\lambda)\alpha.$$

The coefficients are

$$\Delta_0(\lambda) = \lambda \log\left(\frac{(1+e^{-1/\lambda})^2}{4e^{-1/\lambda}}\right),$$

$$\Delta_1(\lambda) = \lambda \log\left(\frac{(1+e^{-1/\lambda})^2}{4e^{-1/\lambda}}\right) + \left(\frac{2e^{-1/\lambda}}{1+e^{-1/\lambda}} - 1\right),$$

$$M_0(\lambda) = \log 2 - H_b\left(\frac{1}{1+e^{-1/(2\lambda)}}\right),$$

$$M_1(\lambda) = -\frac{1}{4\lambda^2}\frac{e^{-1/(2\lambda)}}{(1+e^{-1/(2\lambda)})^2}.$$

where $H_b(p) = -p\log p - (1-p)\log(1-p)$ denotes the binary entropy function.

Case 1: Full interior region

Under the approximations,

$$\Pi^{(1)}(\alpha) = \frac{1}{16} + \frac{2}{3}\frac{\lambda\alpha}{1-\alpha}(A_0 + A_1\alpha + A_2\alpha^2),$$

where

$$A_0 = M_0\Delta_0, \quad A_1 = M_0\Delta_1 + M_1\Delta_0, \quad A_2 = M_1\Delta_1.$$

The first-order condition reduces to the cubic

$$A_2\alpha^3 + (2A_1 - A_2)\alpha^2 + (2A_0 - A_1)\alpha + A_0 = 0.$$

Cardano (1968) introduces a method to find a closed-form solution. In particular, this equation turns out to have exactly one real root $\alpha^{*(1)}(\lambda)$, which falls into $(0,1)$.

Define

$$a(\lambda) = A_2(\lambda), \quad b(\lambda) = 2A_1(\lambda) - A_2(\lambda), \quad c(\lambda) = 2A_0(\lambda) - A_1(\lambda), \quad d(\lambda) = A_0(\lambda).$$

Let

$$p(\lambda) = \frac{3a(\lambda)c(\lambda) - b(\lambda)^2}{3a(\lambda)^2}, \qquad q(\lambda) = \frac{2b(\lambda)^3 - 9a(\lambda)b(\lambda)c(\lambda) + 27a(\lambda)^2 d(\lambda)}{27a(\lambda)^3},$$

and

$$\Delta(\lambda) = \left(\frac{q(\lambda)}{2}\right)^2 + \left(\frac{p(\lambda)}{3}\right)^3.$$

Then the Cardano solution for the optimal advertising intensity withing region 1 is

$$\boxed{\alpha^{*(1)}(\lambda) = -\frac{b(\lambda)}{3a(\lambda)} + \sqrt[3]{-\frac{q(\lambda)}{2} + \sqrt{\Delta(\lambda)}} + \sqrt[3]{-\frac{q(\lambda)}{2} - \sqrt{\Delta(\lambda)}}}$$

where

$$\Delta_0(\lambda) = \lambda \log\left(\frac{(1 + e^{-1/\lambda})^2}{4e^{-1/\lambda}}\right),$$

$$\Delta_1(\lambda) = \lambda \log\left(\frac{(1 + e^{-1/\lambda})^2}{4e^{-1/\lambda}}\right) + \left(\frac{2e^{-1/\lambda}}{1 + e^{-1/\lambda}} - 1\right),$$

$$M_0(\lambda) = \log 2 - H_b\left(\frac{1}{1 + e^{-1/(2\lambda)}}\right), \qquad H_b(p) = -p \log p - (1-p)\log(1-p),$$

$$M_1(\lambda) = -\frac{1}{4\lambda^2}\frac{e^{-1/(2\lambda)}}{(1 + e^{-1/(2\lambda)})^2}.$$

In the main text, we have already derived that the optimal fee in case 1 is $w^{*(1)}(\lambda) = 0.25$

Case 2: Partial interior region

In Case 2, the interior information-acquisition region intersects the support $\varepsilon \sim \mathcal{U}[0,1]$ as $[0,\varepsilon_1]$, i.e. $\varepsilon_0 \leq 0 < \varepsilon_1$. Hence advertising profit integrates mutual information from $\varepsilon = 0$ to ε_1:

$$\Pi_I^{(2)}(\alpha, w) = \frac{\lambda\alpha}{1-\alpha}\int_0^{\varepsilon_1} MI(\varepsilon)\, d\varepsilon.$$

On $[\varepsilon_0,\varepsilon_1]$, approximate

$$MI(\varepsilon) \approx -\frac{4M(\tau)}{(\varepsilon_1-\varepsilon_0)^2}(\varepsilon-\varepsilon_0)(\varepsilon-\varepsilon_1), \qquad \Delta(\tau)=\varepsilon_1-\varepsilon_0.$$

If the lower integration bound is $\varepsilon_a \in [\varepsilon_0,\varepsilon_1]$, define $x_a = \varepsilon_a - \varepsilon_0 \in [0,\Delta]$. Then

$$\int_{\varepsilon_a}^{\varepsilon_1} MI(\varepsilon)\, d\varepsilon = M(\tau)\, f(\Delta,x_a),$$

where

$$f(\Delta,x) = \frac{2}{3}\Delta - \frac{2x^2}{\Delta} + \frac{4x^3}{3\Delta^2}.$$

$\varepsilon_a = 0$, so $x_a(\alpha,w) = -\varepsilon_0(\alpha,w)$.

Recall

$$\varepsilon_0(\alpha,w) = \tau(\alpha)\log\left(\frac{2}{1+e^{-1/\tau(\alpha)}}\right) - w, \qquad \tau(\alpha) = \frac{\lambda}{1-\alpha}.$$

Hence

$$x_a(\alpha,w) = w - \tau(\alpha)\log\left(\frac{2}{1+e^{-1/\tau(\alpha)}}\right).$$

Linearizing around $\alpha = 0$ yields

$$x_a(\alpha,w) \approx (w - \Gamma_0(\lambda)) - \Gamma_1(\lambda)\alpha,$$

where

$$\Gamma_0(\lambda) = \lambda\log\left(\frac{2}{1+e^{-1/\lambda}}\right), \qquad \Gamma_1(\lambda) = \lambda\log\left(\frac{2}{1+e^{-1/\lambda}}\right) + \frac{e^{-1/\lambda}}{1+e^{-1/\lambda}}.$$

Under the introduced approximations

$$\boxed{\Pi_I^{(2)}(\alpha,w) \approx \frac{\lambda\alpha}{1-\alpha}\big(M_0 + M_1\alpha\big)\, f\Big(\Delta_0 + \Delta_1\alpha, (w-\Gamma_0) - \Gamma_1\alpha\Big).}$$

Transaction profit equals

$$\Pi_F^{(2)}(\alpha,w) = \frac{w}{2}\log\left(\frac{\frac{(1-\delta(\alpha))^2}{4\delta(\alpha)}}{(B_w(\alpha,w)-1)\,(\delta(\alpha)B_w(\alpha,w)-1)}\right),$$

where

$$\delta(\alpha) = \exp\left(-\frac{1-\alpha}{\lambda}\right), \qquad B_w(\alpha,w) = \exp\left(\frac{(1-\alpha)w}{\lambda}\right).$$

To express this in closed form, we introduce one final approximation that removes the transcendental dependence of $\Pi_F^{(2)}(\alpha,w)$ on w (induced by the logarithm and $B_w(\alpha,w)$).

Approximation 3 (local polynomial approximation in w around $w_0 = \frac{1}{4}$). Let $w_0 = \frac{1}{4}$ and define $z := w - w_0$. We approximate the Case 2 fee-profit term by a third-order Taylor expansion in w around w_0:

$$\Pi_F^{(2)}(\alpha,w) \approx F_0(\alpha;\lambda) + F_1(\alpha;\lambda)z + \frac{1}{2}F_2(\alpha;\lambda)z^2 + \frac{1}{6}F_3(\alpha;\lambda)z^3,$$

so that

$$\partial_w \Pi_F^{(2)}(\alpha,w) \approx F_1(\alpha;\lambda) + F_2(\alpha;\lambda)z + \frac{1}{2}F_3(\alpha;\lambda)z^2.$$

The derivatives $F_k(\alpha;\lambda)$ are defined by

$$F_k(\alpha;\lambda) := \frac{\partial^k}{\partial w^k}\Pi_F^{(2)}(\alpha,w)\bigg|_{w=w_0}, \qquad k = 0,1,2,3, \qquad w_0 = \tfrac{1}{4},$$

and admit closed form. Write

$$A(\alpha) = \frac{(1-\delta(\alpha))^2}{4\delta(\alpha)}, \qquad b(\alpha) = B_w(\alpha,w_0) = \exp\Big(\frac{(1-\alpha)w_0}{\lambda}\Big), \qquad s(\alpha) = \frac{1-\alpha}{\lambda},$$

and

$$L(\alpha) = \log A(\alpha) - \log\big(b(\alpha)-1\big) - \log\big(\delta(\alpha)b(\alpha)-1\big).$$

Then

$$F_1(\alpha;\lambda) = \frac{1}{2}L(\alpha) - \frac{w_0}{2}s(\alpha)\left(\frac{b(\alpha)}{b(\alpha)-1} + \frac{\delta(\alpha)b(\alpha)}{\delta(\alpha)b(\alpha)-1}\right),$$

$$F_2(\alpha;\lambda) = -s(\alpha)\left(\frac{b(\alpha)}{b(\alpha)-1} + \frac{\delta(\alpha)b(\alpha)}{\delta(\alpha)b(\alpha)-1}\right) + \frac{w_0}{2}s(\alpha)^2\left(\frac{b(\alpha)}{(b(\alpha)-1)^2} + \frac{\delta(\alpha)b(\alpha)}{(\delta(\alpha)b(\alpha)-1)^2}\right),$$

$$F_3(\alpha;\lambda) = \frac{3}{2}s(\alpha)^2\left(\frac{b(\alpha)}{(b(\alpha)-1)^2} + \frac{\delta(\alpha)b(\alpha)}{(\delta(\alpha)b(\alpha)-1)^2}\right) - \frac{w_0}{2}s(\alpha)^3\left(\frac{b(\alpha)(b(\alpha)+1)}{(b(\alpha)-1)^3} + \frac{\delta(\alpha)b(\alpha)\big(\delta(\alpha)b(\alpha)+1\big)}{(\delta(\alpha)b(\alpha)-1)^3}\right)$$

Finally, to maintain consistency with Approximation 2, we linearize these coefficients in α around $\alpha = 0$:

$$F_k(\alpha;\lambda) \approx F_{k0}(\lambda) + F_{k1}(\lambda)\alpha, \qquad F_{k0}(\lambda) = F_k(0;\lambda),\ F_{k1}(\lambda) = \frac{\partial}{\partial\alpha}F_k(\alpha;\lambda)\bigg|_{\alpha=0}.$$

Closed-form policies (Cardano + quadratic). Recall

$$\Delta(\alpha) = \Delta_0 + \Delta_1\alpha, \quad x(\alpha,w) = (w-\Gamma_0) - \Gamma_1\alpha, \quad M(\alpha) = M_0 + M_1\alpha, \quad K(\alpha) = \frac{\lambda\alpha}{1-\alpha}M(\alpha).$$

Then $\Pi_I^{(2)}(\alpha,w) = K(\alpha)\, f(\Delta(\alpha),x(\alpha,w))$ and

$$f_x(\Delta,x) = -\frac{4x}{\Delta} + \frac{4x^2}{\Delta^2}, \qquad f_\Delta(\Delta,x) = \frac{2}{3} + \frac{2x^2}{\Delta^2} - \frac{8x^3}{3\Delta^3}.$$

Using $x(\alpha,w) = x_0(\alpha) + z$ with

$$x_0(\alpha) := (w_0 - \Gamma_0) - \Gamma_1\alpha, \qquad z := w - w_0,$$

we expand f_x as a quadratic in z:

$$f_x(\Delta(\alpha),x_0(\alpha) + z) = A_x(\alpha) + B_x(\alpha)z + C_x(\alpha)z^2,$$

where

$$A_x(\alpha) = -\frac{4x_0(\alpha)}{\Delta(\alpha)} + \frac{4x_0(\alpha)^2}{\Delta(\alpha)^2}, \quad B_x(\alpha) = -\frac{4}{\Delta(\alpha)} + \frac{8x_0(\alpha)}{\Delta(\alpha)^2}, \quad C_x(\alpha) = \frac{4}{\Delta(\alpha)^2}.$$

Step 1 (solve for w given α). The w-FOC

$$\partial_w\Pi_F^{(2)}(\alpha,w) + K(\alpha)\, f_x(\Delta(\alpha),x(\alpha,w)) = 0$$

becomes a quadratic equation in z:

$$a_w(\alpha;\lambda)z^2 + b_w(\alpha;\lambda)z + c_w(\alpha;\lambda) = 0,$$

with coefficients

$$a_w(\alpha;\lambda) = \frac{1}{2}F_3(\alpha;\lambda)+K(\alpha)C_x(\alpha), \quad b_w(\alpha;\lambda) = F_2(\alpha;\lambda)+K(\alpha)B_x(\alpha), \quad c_w(\alpha;\lambda) = F_1(\alpha;\lambda)+K(\alpha)A_x(\alpha).$$

Hence,

$$\boxed{w^{*(2)}(\alpha,\lambda) = w_0 + \frac{-b_w(\alpha;\lambda) \pm \sqrt{b_w(\alpha;\lambda)^2 - 4a_w(\alpha;\lambda)c_w(\alpha;\lambda)}}{2a_w(\alpha;\lambda)}}, \qquad w_0 = \tfrac{1}{4},$$

where the economically relevant branch is the one satisfying $w > 0$ and Case 2 feasibility.

Step 2 (solve for α). Substituting $w^{*(2)}(\alpha,\lambda)$ into the α-FOC,

$$\partial_\alpha\Pi_F^{(2)}(\alpha,w)+K'(\alpha)f(\Delta(\alpha),x(\alpha,w))+K(\alpha)\big(f_\Delta(\Delta(\alpha),x(\alpha,w))\Delta_1-\Gamma_1 f_x(\Delta(\alpha),x(\alpha,w))\big) = 0,$$

and applying the same truncations (linearization in α and the cubic-in-w approximation) yields a univariate cubic equation

$$\boxed{\kappa_3(\lambda)\alpha^3 + \kappa_2(\lambda)\alpha^2 + \kappa_1(\lambda)\alpha + \kappa_0(\lambda) = 0,}$$

where $\kappa_j(\lambda)$ are explicit functions of λ and the approximation coefficients $M_0,M_1,\Delta_0,\Delta_1,\Gamma_0,\Gamma_1,F_{k0},F_{k1}$.

Let $a = \kappa_3(\lambda)$, $b = \kappa_2(\lambda)$, $c = \kappa_1(\lambda)$, $d = \kappa_0(\lambda)$ and define

$$p = \frac{3ac - b^2}{3a^2}, \qquad q = \frac{2b^3 - 9abc + 27a^2 d}{27a^3}, \qquad \Delta_C = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3.$$

Cardano's formula yields the (real) closed-form solution

$$\boxed{\alpha^{*(2)}(\lambda) = -\frac{b}{3a} + \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta_C}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta_C}}},$$

and the economically relevant root is the one in $(0,1)$ satisfying Case 2 feasibility.

Finally,

$$\boxed{w^{*(2)}(\lambda) = w^{*(2)}\big(\alpha^{*(2)}(\lambda),\lambda\big).}$$

B Derivations for Media and Physical Marketplace Extensions

B.1 Media Platform

Under the parabolic approximation,

$$\int_{\varepsilon_0}^{\varepsilon_1} MI(\varepsilon)\, d\varepsilon = \frac{2}{3} M(\tau)\Delta(\tau).$$

Since $w = 0$, advertising profit is

$$\Pi^{\text{med}}(\alpha) = \frac{\lambda\alpha}{1-\alpha} \cdot \frac{2}{3} M(\tau(\alpha))\Delta(\tau(\alpha)).$$

Linearizing around $\alpha = 0$ gives

$$M(\alpha) \approx M_0 + M_1\alpha, \qquad \Delta(\alpha) \approx \Delta_0 + \Delta_1\alpha.$$

Let

$$Q(\alpha) = A_0 + A_1\alpha + A_2\alpha^2.$$

Then

$$\Pi^{\text{med}}(\alpha) = \frac{2}{3}\lambda\frac{\alpha}{1-\alpha}Q(\alpha).$$

Differentiating yields

$$Q(\alpha) + \alpha(1-\alpha)Q'(\alpha) = 0,$$

which expands to the cubic stated in the main text.

B.2 Physical Marketplace

Let

$$\phi(\lambda) = \lambda \log\Big(\frac{2}{1+e^{-1/\lambda}}\Big).$$

Monotonicity. Define

$$g(\lambda) = \log\Big(\frac{2}{1+e^{-1/\lambda}}\Big).$$

Then

$$\phi'(\lambda) = g(\lambda) + \lambda g'(\lambda).$$

Since

$$g'(\lambda) = \frac{e^{-1/\lambda}}{\lambda^2(1+e^{-1/\lambda})} > 0,$$

we have $\phi'(\lambda) > 0$ for all $\lambda > 0$.

Limits.

$$\lim_{\lambda\to 0^+} \phi(\lambda) = 0, \qquad \lim_{\lambda\to\infty} \phi(\lambda) = \log 2.$$

Since $\log 2 \approx 0.693 > 1/4$ and ϕ is strictly increasing, there exists a unique $\lambda^\dagger$ solving

$$\phi(\lambda^\dagger) = \frac{1}{4}.$$

Numerically solving yields

$$\lambda^\dagger \approx 0.463.$$

This completes the threshold characterization.